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CALCULATING THE NOMINAL YIELD POINT OF STRUCTURALLY HETEROGENEOUS POWDERS

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The variety [1, 2] of current methods of calculating the effective nominal yield point $\sigma^*_{0.2}$ of structurally heterogeneous materials, in particular powders, is explained by their hierarchical structure, which gives rise to dissipation of external energy by various components of the structure as a function of the characteristics of the structure of concrete samples and testing conditions. Among the components of the structure of powders, the most important in the formation of their mechanical properties during quasi-equilibrium loading are those of mesoscopic scale, porosity, and concentration heterogeneity. This is shown by the unambiguous link, recorded in experiments, between porosity [3], concentration heterogeneity [4], and mechanical properties. The existence of this link is explained by the fact that the smaller-scale deformations (for example, the intragranular, which in powders are partially or completely blocked by impurity atoms [1]) have the character of small perturbations superimposed on the mesoscopic ones, that is, the energy dissipated by them is small. However, with impact loads the spacial scale of almost all the deformations is much less than the mesoscopic level, and so the energy dissipates on smaller spacial scales, which manifests as multiple formation of foci of failure [5]. Therefore, to construct an adequate method of calculating the effective nominal yield point of a powder in standard tensile tests of samples, it is usually sufficient to consider its structure on the mesolevel.

The method of calculating $\sigma^*_{0.2}$ proposed in this article uses percolation representations,† since with increase in the load applied to the sample, the plastic flow does not encompass the entire volume of the sample immediately, but at first is localized in microvolumes with a decreased concentration of alloy elements and around pores. With increase in the load the volume of the plastic regions increases, and they gradually blend into an infinitely plastic claster, which corresponds to the moment of the appearance of irreversible plastic deformations of the sample as a whole. The infinite elastic claster up to this moment may not be destroyed, and then it will limit the plastic deformation. Gradually, in proportion to the growth of the plastic claster, the elastic claster fails, which permits the development of significant plastic deformations. If during these deformations microcracks form, then as the load increases a third claster will grow — a claster of microcracks, and when they reach a critical size the sample will fail. The proposed model contains the following idealizations: it is assumed that the three-dimensional distribution of the plastic zones around the pores and in the low-alloy regions are uncorrelated, and for the function of the effect of the increase in interparticle contacts in the sintering process on the yield point of the microvolumes, interpolation formula (2) is used.

The relative volume of the material that has gone over into the plastic state has the form

$$\omega = \omega_\vartheta + \omega_\nu - \omega_\vartheta \omega_\nu, \tag{1}$$

where ω_ϑ is the relative volume of the material in the state of plastic deformation due to the concentration of stresses around the pores, while ω_ν is due to heterogeneous alloying. Relation (1) is valid when the formation of plastic zones by the two mechanisms indicated above has an independent and random character. The absence of a three-dimensional correlation between the components of the structure corresponding to them is guaranteed by the independence of the factors determining their formation. Thus, the distribution of pores in the volume of the sample depends on the initial packing of the particles of the powder, and concentration heterogeneity is formed in the process of diffusion mass transfer during sintering [4].

†An analogous approach was proposed in [6] for calculation of the $\sigma^*_{0.2}$ of porous concentration-homogeneous materials.

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TABLE 1. Comparison of Experimental and Computed Values of $\sigma^*_{0.2}$ for 35H2 Steel

ν_{Ni}	$\sigma^*_{0,2}$ (experiment)	$\sigma^*_{0,2}$ (calculation)	Computation Error, %
	MPa		
	Steel based on carbonyl Fe (V_0 = 1.5, B = 0.5)		
0,79	137	135	1
0,60	180	152	16
0,37	207	177	14
0,28	193	195	1
0,20	206	205	1
	Steel based on reduced Fe (V_0 = 1.15, B = 0.7)		
0,98	140	146	4
0,91	150	157	5
0,87	165	159	4
0,69	215	190	12
0,63	250	208	17

TABLE 2. Comparison of Experimental and Computed Values of $\sigma^*_{0.2}$ for 35X Steel

ν_{Ni}	$\sigma^*_{0,2}$ (experiment)	$\sigma^*_{0,2}$ (calculation)	Computation Error, %
	MPa		
	Steel based on carbonyl Fe (V_0 = 2.2, B = 0.5)		
1,30	165	151	8
0,82	190	200	5
0,79	220	205	7
0,77	210	207	2
0,30	230	270	17
0,28	280	288	3
	Steel based on reduced Fe (V_0 = 1.1, B = 0.5)		
0,96	110	133	20
0,65	170	172	1
0,57	185	183	9
0,54	200	188	2
0,52	210	191	4
0,24	230	236	3

From the definition of ω_ϑ it follows that

$$\omega_\vartheta = (1 - \vartheta)(1 - W(\sigma^*)).$$

Here $W(\sigma^*)$ is the fraction of the material going over into the plastic state due to the concentration of stresses around the pores during application of an external load equal to σ^* (the * symbol indicates quantities relating to the entire sample). For $W(\sigma^*)$ we will use an expression obtained and tested experimentally in [5]:

$$W(\sigma^*) = \frac{\vartheta}{1 - \vartheta} \frac{(\sigma^* / < \sigma_{0,2} >)^2}{(1 - \vartheta)^2 (\sigma^* / < \sigma_{0,2} >)^2}$$

($< \sigma_{0.2} >$ is the average value of the yield point of a microvolume not containing pores).

To find ω_v, we will use the results obtained in [4], where it was demonstrated that the distribution of various alloy elements in the volume of the material is independent and logarithmically normal, and its probability density function $\rho(c_1, c_2, \ldots, c_N)$ has the form

$$\rho(c_1, c_2, \ldots, c_N) = \prod_{i=1}^{N} f_i(c_i),$$

TABLE 3. Comparison of Experimental and Computed Values of $\sigma^*_{0.2}$ for 35H2 Steel Based on Carbonyl Fe

ν_{Ni}	ν_{Cr}	$\sigma^*_{0,2}$, MPa (experiment)	$\sigma^*_{0,2}$, MPa (calculation for V_{0Ni} = 1.5, B = 0.62)	Computation Error, %	$\sigma^*_{0,2}$, MPa (calculation for V_{0Cr} = 1.5, B = 0.7)	Computation Error, %
0,78	0,93	274	315	15	321	17
0,48	0,67	453	389	14	391	14
0,38	0,52	435	419	8	433	5
0,30	0,41	478	450	6	469	2
0,16	0,23	499	513	3	533	7

where

$$f_i(c_i) = \exp\left\{-\frac{\ln[c_i\sqrt{(1+V_i^2)}/c_i]^2}{2\ln(1+V_i^2)}\right\}/c_i\sqrt{2\pi\ln(1+V^2)};$$

V_i is the coefficient of variation of the i-th element, equal to the ratio of the root of square of the dispersion of the concentration to the average of its value, and N is the quantity of alloy elements.

We will consider that in the microvolumes of a powder the nominal yield point $\sigma_{0.2}$ depends on chemical composition in the same way as it does in cast materials:

$$\sigma_{0,2} = \alpha(t)\varphi(c_1,c_2,\ldots,c_N).$$

Here $\varphi(c_1, c_2, \ldots, c_N)$ is the dependence of $\sigma_{0.2}$ on concentration in a cast material after the same heat processing; the function $\alpha(t) \leq 1$ takes into account the variation in the strength of interparticle contacts during sintering due to their increase. Let r be the average radius of the powder grains, while x is the radius of the interparticle contact; then $\alpha \sim (x/r)$. Since for small t $\alpha \approx 0$, and $x \sim t^{1/n}$, where $n \in [2, 7]$ and depends on the mechanism of sintering [7], while for large t $\alpha \approx 1$, an interpolation formula having the same asymptotes

$$\alpha = \text{th}(at^{2/n}). \tag{2}$$

is proposed. We will express the homogenization time t through the coefficient of variation of one of the alloy elements, for which we will use the homogenization equation [4]

$$V_i = V_{0i}\exp\left\{-\beta_i\frac{D_i}{r_i^2}\bar{c}_i^{2/3}t\right\}, \tag{3}$$

where D_i, r_i, c_i is the coefficient of mutual diffusion, the average radius of a powder grain, and the average concentration of the i-th component, respectively; β_i, V_{0i} are constants the magnitudes of which are found in independent experiments that study the homogenization of the material on the basis of (3). From (2) and (3) it follows that

$$\alpha = \text{th}\{B_i(\ln(V_{0i}/V_i))^{2/n}\}.$$

The probability distribution density $\sigma_{0.2}$ can be found in the standard way as the distribution of a function of random quantities $c_1, c_2, \ldots, c_N$; for example, for N = 2 [8]

$$g(\sigma_{0,2}) = \int_0^1 f_1(c_1)f_2(c_2(\sigma_{0,2},c_1))\left|\frac{\partial c_2(\sigma_{0,2},c_1)}{\partial\sigma_{0,2}}\right|dc_1. \tag{4}$$

In (4) there are three parameters: V_{0i}, V_i and B_i. The values of V_{0i}, V_i depend on the characteristics of the initial powder and the sintering modes and can be determined in accordance with [4]. Therefore B_i is the only parameter of the method.

In many metal-powder systems the alloying has a weak effect on the elastic constants. Therefore, under the condition of a homogeneous external load it may be assumed that the distribution of stresses in the microvolumes does not depend on the distribution of concentrations, i.e. is homogeneous. Let a stress σ^* be applied to the material; then the fraction of microvolumes shifting to the plastic state is

$$\omega_v = (1 - \vartheta) \int_{\sigma_{min}}^{\sigma^*} g(y)dy,$$

where σ_{min} is the minimum value of $\sigma_{0.2}$ (usually $\sigma_{min} = \alpha\varphi(0.0)$).

In percolation theory it has been shown that in the three-dimensional case of the problem of flow through nodes, the claster becomes infinite (permeates the entire material), when its volumetric fraction reaches the value of 0.17. Therefore the condition of failure of an elastic claster constraining a macroscopic plastic deformation has the form

$$0{,}17 = (1 - \vartheta)(1 - \omega) = (1 - \vartheta)(1 - \omega_\vartheta - \omega_v + \omega_\vartheta\omega_v). \quad (5)$$

Substituting into (5) the expressions for ω_ϑ and ω_v in which σ^* is replaced by $\sigma^*_{0.2}$, we obtain an integral equation for finding $\sigma^*_{0.2}$.

Tables 1-3 give the results of an experimental test of the proposed method. The samples were pressed at 780 MPa in a two-sided pattern, sintered in a range of modes from 1150°/1h to 1300°/6h. Mechanical tests were carried out in accordance with GOST 1793-73. The solution of equation (5) was found numerically, setting n = 5 for steels based on carbonyl Fe, which corresponds to the oversintering of the grains by the mechanism of volumetric diffusion in the presence of drainage in the link between the particles of the powder, and n = 4 for steels based on reduced iron, contraction in which is controlled by boundary diffusion [9]. We found that function (5) approximates the experimental data with an error of the order of the experimental error.

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